1. Positive integers \( a < b \) are given. Prove that among every \( b \) consecutive positive integers there are two numbers whose product is divisible by \( ab \).  

(S. Berlov)

2. Two polynomials  
   
   \[
   f(x) = a_{100}x^{100} + a_{99}x^{99} + \ldots + a_1x + a_0
   \]
   
   and
   
   \[
   g(x) = b_{100}x^{100} + b_{99}x^{99} + \ldots + b_1x + b_0
   \]
   
   of degree 100 differ from each other by a permutation of coefficients. It is known that \( a_i \neq b_i \) for \( i = 0, 1, 2, \ldots, 100 \). Is it possible that \( f(x) \geq g(x) \) for all real \( x \)?  

(A. Golovanov)

3. \( AA_1, BB_1, CC_1 \) are altitudes of an acute triangle \( ABC \). A circle passing through \( A_1 \) and \( B_1 \) touches the arc \( AB \) of its circumcircle at \( C_2 \). The points \( A_2, B_2 \) are defined similarly. Prove that the lines \( AA_2, BB_2, CC_2 \) are concurrent.  

(R. Sakhipov)

4. Determine maximum real \( k \) such that there exist a set \( X \) and its subsets \( Y_1, Y_2, \ldots, Y_{31} \) satisfying the following conditions:
   
   (1) for every two elements of \( X \) there is an index \( i \) such that \( Y_i \) contains neither of these elements;
   
   (2) if any non-negative numbers \( \alpha_i \) are assigned to the subsets \( Y_i \) and \( \alpha_1 + \ldots + \alpha_{31} = 1 \) then there is an element \( x \in X \) such that the sum of \( \alpha_i \) corresponding to all the subsets \( Y_i \) that contain \( x \) is at least \( k \).  

(I. Bogdanov, G. Chelnokov)

Second day

5. What minimum number of colours is sufficient to colour all positive real numbers so that every two numbers whose ratio is 4 or 8 have different colours?  

(A. Golovanov)

6. Point \( D \) is chosen on the side \( AB \) of triangle \( ABC \). Point \( L \) inside the triangle \( ABC \) is such that \( BD = LD \) and \( \angle LAB = \angle LCA = \angle DCB \). It is known that \( \angle ALD + \angle ABC = 180^\circ \). Prove that \( \angle BLC = 90^\circ \).  

(R. Sakhipov)

7. Several knights are arranged on an infinite chessboard. No square is attacked by more than one knight (in particular, a square occupied by a knight can be attacked by one knight but not by two). Sasha outlined a \( 14 \times 16 \) rectangle. What maximum number of knights can this rectangle contain?  

(S. Berlov)

8. Prove that there exists a positive \( c \) such that for every positive integer \( N \) among any \( N \) positive integers not exceeding \( 2N \) there are two numbers whose greatest common divisor is greater than \( cN \).  

(F. Petrov)